VECTOR ALGEBRA

Time: 2.00 Hrs. **Max. Marks: 200**

Note:

This question paper contains 50 questions. Each question has four alternatives, out of four one is correct. For each correct answers 4 marks will be given and for each wrong answer 1 mark will be deducted.

1. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ given by:

   (a) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{a^2}$ $\frac{\vec{b}\times(\vec{a}\times\vec{b})}{b^2}$ (c) $\frac{(\vec{a}\cdot\vec{b})\vec{b}}{a^2}$ (d) $\frac{(\vec{b}\cdot\vec{a})\vec{a}}{a^2}$ [ ]

2. The equation of the plane that is tangent to the surface $xyz = 8$ at the point (1, 2, 4) is

   (a) $x + 2y + 4z = 12$ (b) $4x + 2y + z = 12$

   (c) $x + 4y + 2z = 12$ (d) $x + y + z = 7$ [ ]

3. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is

   (a) $\hat{\imath} + \hat{\jmath}$ (b) $\hat{\jmath} + \hat{k}$ (c) $\hat{\imath} + \hat{\jmath} + \hat{k}$ (d) $2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ [ ]

4. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is

   (a) $\frac{bc\hat{\imath}+ca\hat{\jmath}+ab\hat{k}}{\sqrt{b^2c^2+c^2a^2+a^2b^2}}$ (b) $\frac{a\hat{\imath}+b\hat{\jmath}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

   (c) $\frac{b\hat{\imath}+c\hat{\jmath}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$ [ ]

5. A unit vector $\hat{n}$ on the xy-plane is at an angle of 120° with respect to $\hat{\imath}$. The angle between the vectors $\vec{u} = a\hat{\imath} + b\hat{n}$ and $\vec{v} = a\hat{n} + b\hat{\imath}$ will be 60° if

   (a) $b = \sqrt{3}a/2$ (b) $b = 2a/\sqrt{3}$ (c) $b = a/2$ (d) $b = a$ [ ]

6. If $\vec{A} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$, then the integral $\oint_C \vec{A}\cdot d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by $x = 0$, $x = a$ and $y = 0$, $y = b$) is:

   (a) $\frac{1}{2}(a^3 + b^3)$ (b) $\pi(ab^2 + a^2b)$ (c) $\pi(a^3 + b^3)$ (d) 0 [ ]

7. If $\vec{A} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ and C is the circle of unit radüus in the plane defined by $z = 1$, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}\cdot d\vec{l}$ is:

   (a) π / 2 (b) π (c) π / 4 (d) 0 [ ]

8. Let $\vec{r}$ denote the position vector of any point in three-dimensional space and $r = |\vec{r}|$. Then
   (a) $\nabla \cdot \vec{r} = 0$ and $\nabla \times \vec{r} = \vec{r}/r$ (b) $\nabla \cdot \vec{r} = 0$ and $\nabla^2 \vec{r} = 0$
   (c) $\nabla \cdot \vec{r} = 3$ and $\nabla^2 \vec{r} = \vec{r}/r^2$ (b) $\nabla \cdot \vec{r} = 3$ and $\nabla \times \vec{r} = 0$

9. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the positive vector, then the value of the $\iint_S \vec{r} \cdot \hat{n}\, ds$ is:
   (a) V (b) $2V$ (c) 0 (d) $3V$ [ ]

10. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$
    (a) the three vectors are orthonormal
    (b) the three vectors are linearly independent
    (c) the three vectors cannot form a basis in a three-dimensional real vector space
    (d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as the linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$
    [ ]

11. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2 \vec{A}$ is:
    (a) 1 (b) 3 (c) 0 (d) –3 [ ]

12. Which of the following vectors is orthogonal to the vector $(a\hat{\imath} + b\hat{\jmath})$, where a and b ($a \neq b$) are constants, and $\hat{\imath}$ and $\hat{\jmath}$ are unit orthogonal vectors?
    (a) $-b\hat{\imath} + a\hat{\jmath}$ (b) $-a\hat{\imath} + b\hat{\jmath}$ (c) $-a\hat{\imath} - b\hat{\jmath}$ (d) $-b\hat{\imath} - a\hat{\jmath}$ [ ]

13. A vector $\vec{A} = (5x + 2y)\hat{\imath} + (3y - z)\hat{\jmath} + (2x - az)\hat{k}$ is solenoidal, if the constant a has a value:
    (a) 4 (b) –4 (c) 8 (d) –8 [ ]

14. The unit vector normal to the surface $3x^2 + 4y = z$ at the point (1, 1,7) is:
    (a) $(-6\hat{\imath} + 4\hat{\jmath} + \hat{k})/\sqrt{53}$ (b) $(4\hat{\imath} + 6\hat{\jmath} - \hat{k})/\sqrt{53}$
    (c) $(6\hat{\imath} + 4\hat{\jmath} - \hat{k})/\sqrt{53}$ (d) $(4\hat{\imath} + 6\hat{\jmath} + \hat{k})/\sqrt{53}$ [ ]

15. The two vectors $\vec{P} = \hat{\imath}$ and $\vec{Q} = (\hat{\imath} + \hat{\jmath})/\sqrt{2}$ are
    (a) related by a rotation (b) related by a reflection through the xy-plane
    (c) related by an inversion (d) not linearly independent [ ]

16. The curl of the vector $\vec{A} = z\hat{\imath} + x\hat{\jmath} + y\hat{k}$ is given by:
    (a) $\hat{\imath} + \hat{\jmath} + \hat{k}$ (b) $\hat{\imath} - \hat{\jmath} + \hat{k}$
    (c) $\hat{\imath} + \hat{\jmath} - \hat{k}$ (d) $-\hat{\imath} - \hat{\jmath} - \hat{k}$ [ ]

17. Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$. The surface integral of this vector over the surface of a cube of size and centred at the origin is:
    (a) 0 (b) 2π (c) $2\pi a^3$ (d) 4π [ ]

18. Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$. Which one of the following is not correct ?
    (a) Value of the line integral of this vector around any closed curve is zero.
    (b) This vector can be written as the gradient of some scalar function.
    (c) The line integral of this vector from point P to Q is independent of the path taken.
    (d) This vector can represent the magnetic field of some current distribution.
    [ ]

19. For the function $\varphi = x^2y + xy$, the value of $|\nabla\varphi|$ at $x = y = 1$ is:
    (a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$ [ ]

20. The unit normal to the curve $x^3y^2 + xy = 17$ at the point (2, 0) is:
    (a) $\frac{(\hat{\imath}+\hat{\jmath})}{\sqrt{2}}$ (b) $-\hat{\imath}$ (c) $-\hat{\jmath}$ (d) $\hat{\jmath}$ [ ]

21. If a vector field $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}$, then $\nabla \times (\nabla \times \vec{F})$ is:
    (a) 0 (b) $\hat{\imath}$ (c) $2\hat{\jmath}$ (d) $3\hat{k}$ [ ]

22. Given the four vectors $u_1 = \begin{pmatrix}1\\2\\1\end{pmatrix}, u_2 = \begin{pmatrix}3\\-5\\1\end{pmatrix}, u_3 = \begin{pmatrix}2\\4\\-8\end{pmatrix}$ and $u_4 = \begin{pmatrix}3\\6\\-12\end{pmatrix}$. The linearly dependent pair is:
    (a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4 [ ]

23. The value of $\oint_S \frac{\vec{r}\cdot d\vec{s}}{r^3}$, where $\vec{r}$ is the position vector and S is the closed surface enclosing the origin, is
    (a) zero (b) π (c) 4π (d) 8π [ ]

24. If $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, then
    (a) $\nabla\cdot\vec{r} = 0$ and $\nabla|\vec{r}| = \vec{r}$ (b) $\nabla\cdot\vec{r} = 2$ and $\nabla|\vec{r}| = \vec{r}$
    (c) $\nabla\cdot\vec{r} = 2$ and $\nabla|\vec{r}| = \frac{\vec{r}}{r}$ (d) $\nabla\cdot\vec{r} = 3$ and $\nabla|\vec{r}| = \frac{\vec{r}}{r}$ [ ]

25. The curl of the vector field $\vec{F}$ is $2\hat{x}$. Identify the appropriate vector field $\vec{F}$ from the choices given below:
    (a) $\vec{F} = 2x\hat{y} + 5y\hat{z}$ (b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$
    (c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$ (d) $\vec{F} = 2x\hat{y} + 5y\hat{z}$ [ ]

26. The value of contour integral $\left|\int_C \vec{r} \times d\vec{\theta}\right|$, for a circle C of radius r with centre at the origin is
    (a) $2\pi r$ (b) $r^2 / 2$ (c) πr^2 (d) r [ ]

27. Consider the set of vectors in three-dimensional real vector space R^3, $S = \{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$. Which of the following statement is true?
    (a) S is not a linearly independent set
    (b) S is a basis for R^3
    (c) The vectors in S are orthogonal

(d) An orthogonal set of vectors cannot be generated from S

28. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that:
(a) $\nabla \times \vec{F} = 0$ (b) $\nabla \cdot \vec{F} = 0$ (c) $\nabla V = 0$ (d) $\nabla^2 V = 0$ [ ]

29. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point $P(1, 1, 1)$ is:
(a) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$ [ ]

30. Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector and $\hat{n}$ be a unit vector normal to the surface. The surface integral $\int_S \vec{r} \cdot \hat{n}\, ds$ over the closed surface of the cylinder is:
(a) $2\pi a^2(a + h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) zero [ ]

31. Identify the correct statement for the vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath}$
(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent
(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent
(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal
(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized [ ]

32. For a scalar function ϕ satisfying the Laplace equation, $\nabla\phi$ has
(a) zero curl and non-zero divergence
(b) non-zero curl and zero divergence
(c) zero curl and zero divergence
(d) non-zero curl and non-zero divergence [ ]

33. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\nabla[\vec{A} \cdot (\vec{B} \times \vec{r})]$ is
(a) $\vec{A} \cdot \vec{B}$ (b) $\vec{A} \times \vec{B}$ (b) $\vec{r}$ (d) zero [ ]

34. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point $(1, 1, 1)$ is
(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$ [ ]

35. Four forces are given below in Cartesian and spherical polar coordinates.
(i) $\vec{F}_1 = K\exp\left(-\frac{r^2}{R^2}\right)\hat{r}$ (ii) $\vec{F}_2 = K(x^3\hat{y} - y^3\hat{z})$
(iii) $\vec{F}_3 = K(x^3\hat{x} + y^3\hat{y})$ (iv) $\vec{F}_4 = K(\hat{\varphi}/r)$

where K is a constant. Identify the correct option.
(a) (iii) and (iv) are conservative but (i) and (ii) are not
(b) (i) and (ii) are conservative but (iii) and (iv) are not
(c) (ii) and (iii) are conservative but (i) and (iv) are not
(d) (i) and (iii) are conservative but (ii) and (iv) are not [ ]

36. The direction of ∇f for a scalar field $f(x,y,z)=\frac{1}{2}x^2-xy+\frac{1}{2}z^2$ at the point P (1, 1, 2) is:

(a) $\frac{-\hat{j}-2\hat{k}}{\sqrt{5}}$ (b) $\frac{-\hat{j}+2\hat{k}}{\sqrt{5}}$ (c) $\frac{\hat{j}-2\hat{k}}{\sqrt{5}}$ (d) $\frac{\hat{j}+2\hat{k}}{\sqrt{5}}$ [ ]

37. The work done by a force in moving a particle of mass m from any point (x, y) to a neighbouring point $(x+dx, y+dy)$ is given by $dW=2xydx+x^2dy$. The work done for a complete cycle around a unit circle is:

(a) 0 (b) 1 (c) 3 (d) 2π [ ]

38. The equation of a surface of revolution is $z=\pm\sqrt{\frac{3}{2}x^2+\frac{3}{2}y^2}$. The unit vector normal to the surface at the point $A\left(\sqrt{\frac{3}{2}},0,1\right)$ is

(a) $\sqrt{\frac{3}{5}}\hat{\imath}+\frac{2}{\sqrt{10}}\hat{k}$ (b) $\sqrt{\frac{3}{5}}\hat{\imath}-\frac{2}{\sqrt{10}}\hat{k}$

(c) $\sqrt{\frac{3}{5}}\hat{\imath}+\frac{2}{\sqrt{5}}\hat{k}$ (d) $\sqrt{\frac{3}{10}}\hat{\imath}+\frac{2}{\sqrt{10}}\hat{k}$ [ ]

39. The line integral $\int_A^B \vec{F}\cdot\vec{dl}$, where $\vec{F}=\frac{x}{\sqrt{x^2+y^2}}\hat{x}+\frac{y}{\sqrt{x^2+y^2}}\hat{y}$, along the semi-circular path as shown in the figure below is:

(a) –2 (b) 0 (c) 2 (d) 4 [ ]

40. If $\vec{F}$ is a constant vector and $\vec{r}$ is the position vector then $\nabla(\vec{F}\cdot\vec{r})$ would be

(a) $(\nabla\cdot\vec{r})\vec{F}$ (b) $\vec{F}$ (c) $(\nabla\cdot\vec{F})\vec{r}$ (d) $|\vec{r}|\,\vec{F}$ [ ]

41. For vectors $\vec{a}=\hat{\jmath}+\hat{k}, \vec{b}=2\hat{\imath}+3\hat{\jmath}-5\hat{k}$ and $\vec{c}=\hat{\jmath}-\hat{k}$, the vector product $\vec{a}\times(\vec{b}\times\vec{c})$ is

(a) in the same direction as $\vec{c}$

(b) in the direction opposite to $\vec{c}$

(c) in the same direction as $\vec{b}$

(d) in the direction opposite to $\vec{b}$ [ ]

42. If the surface integral of the field $\vec{A}(x,y,z)=2\alpha x\hat{\imath}+\beta y\hat{\jmath}-3\gamma z\hat{k}$ over the closed surface of an arbitrary unit sphere is to be zero, then the relationship between α, β and γ is:

(a) $\alpha+\frac{\beta}{6}-\gamma=0$ (b) $\frac{\alpha}{3}+\frac{\beta}{6}-\frac{\gamma}{2}=0$

(c) $\frac{\alpha}{2}+\beta-\frac{\gamma}{3}=0$ (d) $\frac{2}{\alpha}+\frac{1}{\beta}-\frac{3}{\gamma}=0$ [ ]

43. The line integral $\oint \vec{A} \cdot \vec{dl}$ of a vector field $\vec{A}(x,y) = \frac{1}{r^2}(-y\hat{\imath} + x\hat{\jmath})$, where $r^2 = x^2 + y^2$, is taken around a square (see figure) of side unit length and centered at (x_0, y_0) with $|x_0| > \frac{1}{2}$ and $|y_0| > \frac{1}{2}$. If the value of the integral is L, then
(a) L depends on (x_0, y_0)
(b) L is independent of (x_0, y_0) and its value is –1
(c) L is independent of (x_0, y_0) and its value is 0
(d) L is independent of (x_0, y_0) and its value is 2 [ ]

44. Consider a vector field $\vec{F} = y\hat{\imath} + xz^3\hat{\jmath} - yz\hat{k}$. Let C be the circle $x^2 + y^2 = 4$ on the plane $z = 2$, oriented counter-clockwise. The value of contour integral $\oint_C \vec{F} \cdot d\vec{r}$ is
(a) 28 π (b) 4 π (c) – 4 π (d) –28 π [ ]

45. A particle of mass m is moving in xy-plane. At any given time t, its position vector is given by $\vec{r}(t) = A\cos\omega t\,\hat{\imath} + B\sin\omega t\,\hat{\jmath}$, where A, B and ω are constants with $A \neq B$. Which of the following statements are true?
(a) orbit of the particle is a circle
(b) speed of the particle is constant
(c) at any given time t, the particle experiences a force away from the origin
(d) the angular momentum of the particle is $m\omega AB\hat{k}$ [ ]

46. The tangent line to the curve $x^2 + xy + 5 = 0$ at (1, 1) is represented by
(a) $y = 3x - 2$ (b) $y = -3x + 4$
(c) $x = 3y - 2$ (d) $x = -3y + 4$ [ ]

47. Consider a closed triangular contour traversed in counter-clockwise direction, as shown in the figure. The value of the integral, $\oint \vec{F} \cdot d\vec{l}$ evaluated along this contour, for a vector field, $\vec{F} = y\hat{e}_x - x\hat{e}_y$, is
($\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$, are unit vectors in Cartesian - coordinate system).
(a) 2 (b) –2 (c) 4 (d) –4 [ ]

48. A hemispherical shell is placed on the xy-plane centered at the origin. For a vector field $\vec{E} = (-y\hat{e}_x + x\hat{e}_y)/(x^2 + y^2)$, the value of the integral $\int_S (\nabla \times \vec{E}) \cdot d\vec{s}$ over the hemispherical surface is:
($d\vec{s}$ is the elemental surface area, $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are unit vectors in Cartesian-coordinate system).
(a) 2π (b) 4π (c) 8π (d) 16π [ ]

49. A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$. The components of its acceleration at $t = 1$ in the direction $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$, is equal to:
(a) $2/\sqrt{14}$ (b) $-2/\sqrt{14}$ (c) $16/\sqrt{14}$ (c) $-16/\sqrt{14}$ [ ]

50. The value of line integral $\int grad\,(x + y + z) \cdot d\vec{r}$ from (0, 1, –1) to (1, 2, 0) is
(a) –1 (b) 3 (c) 2 (d) 0 [ ]